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Hamiltonian analyses of new set of dual actions of bosonic p-branes

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Abstract: We make hamiltonian analyses of the new set of dual bosonic p-brane (including string) actions which contain high non-linearity. The difficulties exist in two basic steps of the hamiltonian procedure, that is, in calculating canonical momenta and in solving velocities in terms of momenta. The former difficulty can be overcome by an ADM reparametrization of induced metrics, while the latter may be circumvented by some modification to the usual Hamiltonian procedure. We also compare our results with that of the other set composed of known dual p-brane actions.

Keywords: Field Theories in Higher Dimensions, p-branes.

Contents

1. Introduction

Bosonic p-branes [1] are extended objects that are embedded in a higher dimensional space-time, and their dynamics is governed in general by two sets of dual actions [2] in which they are symbolized by (S_{NG}, S_P, $S_W^{\rm I}$, $S_W^{\rm II}$) and (S_{NG}, A_P, $A_W^{\rm I}$, $A_W^{\rm II}$). The first set consists of four known actions, while the second contains three new ones where A_P is not Weyl-invariant but $A_W^{\rm I}$ and $A_W^{\rm II}$ are Weyl-invariant. The seven different formulations of actions are classically equivalent from the point of view that they lead to the same equations of motion. Each set is closed by dualization with respect to various field variables appeared in actions. The second, i.e. the new set is discovered by using the paraent or master action approach [3] together with a special proposal of parent actions. (For details of derivation and discussion of duality, see reference [2].)

In the first set of dual actions, the Nambu-Goto action S_{NG} is proportional to a $(p+1)$-dimensional worldvolume by Nambu [4] and Goto [5] for a string ($p=1$). S_P is the p-brane action with an auxiliary worldvolume metric and a cosmological term, where the subscript means p-branes, whose formulation of a membrane ($p=2$) was proposed by Howe and Tucker [6] in the construction of a locally supersymmetric invariant model of a spinning membrane. Quite noticeable is that S_P is not Weyl-invariant for the general cases, $p \geq 2$, but Weyl-invariant for the string theory, i.e. $p=1$. This string action was first given by Brink, Di Vecchia and Howe and Deser and Zumino [7, 8], and its virtues in path integral quantization were emphasized by Polyakov [9]. For p-brane ($p \geq 2$) cases a first order form of Weyl-invariant actions[1] was first established [10] from the Nambu-Goto action through directly replacing the induced metric by a Wely-invariant combination of

[1]This action is not discussed in our paper because it cannot be classified into any of the two sets of dual actions. For details, see reference [2].

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the worldvolume metric and the induced metric itself. $S_W^{\rm I}$ and $S_W^{\rm II}$ called in reference [2] first and second Weyl-invariant actions respectively, possess a Weyl symmetry. They were constructed [11, 12, 13, 1] partly because of the important role played by the Weyl symmetry in covariant quantization of strings.

In the new set of dual actions, except for the Nambu-Goto action which appears in both sets, the other three actions are newly constructed [2]. A_P which corresponds to S_P does not possess the Weyl invariance *even* for the string theory, while $A_W^{\rm I}$ and $A_W^{\rm II}$ called in reference [2] first and second *new* Weyl-invariant actions respectively, are thus proposed with the aim at recovering such a symmetry. In particular, the three new actions contain high non-linearity of derivatives of spacetime with respect to worldvolume parameters which originates from a special definition of contravariant components of induced metrics. They have the following forms:

$$A_P = -\frac{T}{2}\int d^{p+1}\xi\sqrt{-g}\left[-g_{ij}h^{ij} + (p+3)\right], \tag{1.1}$$

$$A_W^{\rm I} = -\frac{T}{2}\int d^{p+1}\xi\sqrt{-g}\left[-\Phi^{(p+3)/2}g_{ij}h^{ij} + (p+3)\Phi^{(p+1)/2}\right], \tag{1.2}$$

and

$$A_W^{\rm II} = -T\int d^{p+1}\xi\sqrt{-g}\left(\frac{1}{p+1}g_{ij}h^{ij}\right)^{-(p+1)/2}, \tag{1.3}$$

where T is the p-brane tension. We have utilized the same notation as that in reference [2] and, for the purpose of explicitness, rewrite it as follows:

$$\eta_{\mu\nu} = {\rm diag}(-1, 1, \ldots, 1), \tag{1.4}$$

is the flat metric of a D-dimensional Minkowski spacetime. Greek indices $(\mu, \nu, \sigma, \cdots)$ run over $0, 1, \ldots, D-1$.

$$h_{ij} \equiv \frac{\partial X^\mu}{\partial \xi^i}\frac{\partial X^\nu}{\partial \xi^j}\eta_{\mu\nu}, \tag{1.5}$$

is defined as the induced metric in the $(p+1)$-dimensional worldvolume spanned by $p+1$ arbitrary parameters ξ^i. Latin indices $(i, j, k, \ldots)$ take the values $0, 1, \ldots, p$. Coordinates $X^\mu(\xi^i)$ are a D-component Lorentz vector field in the spacetime and D scalar fields in the worldvolume as well. g_{ij} stands for the auxiliary worldvolume metric, g^{ij} its inverse, and $g \equiv \det(g_{ij})$. $\Phi(\xi^i)$ is introduced as a scalar in both the spacetime and worldvolume in order to keep equation (1.2) invariant under the Lorentz transformation and reparametrization. Especially, note that h^{ij} is not $g^{ik}g^{jl}h_{kl}$, but is defined as the inverse of h_{ij} and is independent of g_{ij}. With such a definition and equation (1.5), we can see that there exist highly non-linear terms composed of $\frac{\partial X^\mu}{\partial \xi^i}\frac{\partial X^\nu}{\partial \xi^j}\eta_{\mu\nu}$ in h^{ij}.

In this paper we make hamiltonian analyses for A_P, $A_W^{\rm I}$, and $A_W^{\rm II}$, respectively. Note that the hamiltonian analysis (including also an auxiliary field) was applied to [14] Weyl-invariant strings, D-strings, and to general relativity, and that the applications to the manifestly diffeomorphism invariant actions of the models were made for the first time. The first problem we encounter is how to re-formulate the actions A_P, $A_W^{\rm I}$, and $A_W^{\rm II}$ explicitly in terms of $\frac{\partial X^\mu}{\partial \xi^i}\frac{\partial X^\nu}{\partial \xi^j}\eta_{\mu\nu}$, that is, to compute the inverse of h_{ij}. This can be resolved

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by applying an ADM reparametrization [15] to the induced metric. Incidentally, similar application to the auxiliary worldvolume metric appeared in references [16, 11]. After that we calculate canonical momenta. If we follow the usual hamiltonian procedure, the second problem might be substitution of velocities by the momenta. However, because of the high non-linearity mentioned above to fulfill such a substitution is almost impossible. The same case has occurred [17] when dealing with S_W^{II} whose non-linearity, less complicated than ours, is just the expedient of $\frac{\partial X^\mu}{\partial \xi^i}\frac{\partial X^\nu}{\partial \xi^j}\eta_{\mu\nu}$ to the power $(p+1)/2$. In reference [17] some modification to the usual hamiltonian procedure is suggested in order to circumvent the difficult task of replacing velocities by momenta in hamiltonian densities. We find that this modification is suitable to be applied in a wider region of including our new actions. In the next section we make a brief review of the modified hamiltonian procedure. Following this procedure, we then canonically analyse A_P, A_W^{I}, and A_W^{II} in sections 3, 4 and 5, respectively. At last, in section 6 we will make a summary and compare our results with that of the first set composed of known dual p-brane actions.

2. Brief review of modified hamiltonian procedure

In order for this paper to be self-contained we simply repeat the main context of the modified hamiltonian procedure [17], but use our notation for the sake of consistency in the paper as a whole.

Consider a general system described by the action

$$S = \int d^{p+1}\xi \; \mathcal{L}(\Psi^A, \partial_i \Psi^A)\,, \tag{2.1}$$

where Ψ^A represent all field variables with index A taking values of number of the variables, and $\partial_i \Psi^A$ stand for $\frac{\partial \Psi^A}{\partial \xi^i}$, $i = 0, 1, \ldots, p$. The canonical momenta conjugate to Ψ^A are defined by

$$P_A \equiv \frac{\partial \mathcal{L}}{\partial \dot{\Psi}^A}\,, \tag{2.2}$$

where $\dot{\Psi}^A \equiv \partial_0 \Psi^A$. If the Hessian matrix is singular, there are, say, R primary constraints

$$\Omega_M = \Omega_M(\Psi^A, P_A)\,, \qquad M = 1, \ldots, R\,, \tag{2.3}$$

which come from the definition equation (2.2). The primary hamiltonian density that generates evolution of the system takes the form

$$\mathcal{H} = P_A \dot{\Psi}^A - \mathcal{L} + \lambda^M \Omega_M\,, \tag{2.4}$$

where λ^M are auxiliary Lagrange multipliers introduced.

The usual procedure at this point is trying to replace velocities $\dot{\Psi}^A$ by momenta P_A. Instead, one computes the general variation of $\mathcal{H}$

$$\delta\mathcal{H} = \delta P_A \dot{\Psi}^A + P_A \delta\dot{\Psi}^A + \delta\lambda^M \Omega_M + \lambda^M \delta\Omega_M - \delta\mathcal{L}\,. \tag{2.5}$$

In terms of the expressions

$$
\begin{aligned}
\delta\Omega_M &= \frac{\partial\Omega_M}{\partial\Psi^A}\delta\Psi^A + \frac{\partial\Omega_M}{\partial P_A}\delta P_A\,,\\
\delta\mathcal{L} &= \left(\frac{\partial\mathcal{L}}{\partial\Psi^A} - \partial_a\frac{\partial\mathcal{L}}{\partial(\partial_a\Psi^A)}\right)\delta\Psi^A + \frac{\partial\mathcal{L}}{\partial\dot{\Psi}^A}\delta\dot{\Psi}^A\,,
\end{aligned}
\tag{2.6}
$$

one thus arrives at the formulation of the general variation

$$
\begin{aligned}
\delta\mathcal{H} = &\left(P_A - \frac{\partial\mathcal{L}}{\partial\dot{\Psi}^A}\right)\delta\dot{\Psi}^A + \left(\dot{\Psi}^A + \lambda^M\frac{\partial\Omega_M}{\partial P_A}\right)\delta P_A +\\
&+\left(\lambda^M\frac{\partial\Omega_M}{\partial\Psi^A} - \frac{\partial\mathcal{L}}{\partial\Psi^A} + \partial_a\frac{\partial\mathcal{L}}{\partial(\partial_a\Psi^A)}\right)\delta\Psi^A + \Omega_M\delta\lambda^M\,,
\end{aligned}
\tag{2.7}
$$

where indices $a, b, \ldots, f$ take values $1, 2, \ldots, p$ here and in the following sections. From the above formulation together with equation (2.2) one can obtain the generalized hamiltonian equations

$$
\begin{aligned}
\frac{\partial\mathcal{H}}{\partial\dot{\Psi}^A} &= P_A - \frac{\partial\mathcal{L}}{\partial\dot{\Psi}^A} = 0\,,\\
\frac{\partial\mathcal{H}}{\partial P^A} &= \dot{\Psi}^A + \lambda^M\frac{\partial\Omega_M}{\partial P_A}\,,\\
\frac{\partial\mathcal{H}}{\partial\Psi^A} &= \lambda^M\frac{\partial\Omega_M}{\partial\Psi^A} - \frac{\partial\mathcal{L}}{\partial\Psi^A} + \partial_a\frac{\partial\mathcal{L}}{\partial(\partial_a\Psi^A)}\,,\\
\frac{\partial\mathcal{H}}{\partial\lambda^M} &= \Omega_M\,.
\end{aligned}
\tag{2.8}
$$

The next step in the hamiltonian procedure is to impose the time conservation of the primary constraints Ω_M. This can also be done without replacing velocities by momenta. The time evolution of constraints takes the form

$$
\dot{\Omega}_M(\xi^0, \boldsymbol{\xi}) = \int d^p\boldsymbol{\xi}'\{\Omega_M(\xi^0, \boldsymbol{\xi}), \mathcal{H}(\xi^0, \boldsymbol{\xi}')\} \approx 0\,,
\tag{2.9}
$$

where "$\approx$" means Dirac's weak equality [18]. By using the derivative properties of Poisson brackets

$$
\{\Omega_M, \mathcal{L}\} = \{\Omega_M, \Psi^A\}\frac{\partial\mathcal{L}}{\partial\Psi^A} + \{\Omega_M, \dot{\Psi}^A\}\frac{\partial\mathcal{L}}{\partial\dot{\Psi}^A} + \{\Omega_M, \partial_a\Psi^A\}\frac{\partial\mathcal{L}}{\partial(\partial_a\Psi^A)}\,,
\tag{2.10}
$$

and equation (2.4) one calculates the Poissson bracket between Ω_M and $\mathcal{H}$

$$
\begin{aligned}
\{\Omega_M, \mathcal{H}\} = &\{\Omega_M, \dot{\Psi}^A\}\left(P_A - \frac{\partial\mathcal{L}}{\partial\dot{\Psi}^A}\right) + \{\Omega_M, P_A\}\dot{\Psi}^A - \{\Omega_M, \Psi^A\}\frac{\partial\mathcal{L}}{\partial\Psi^A} -\\
&-\{\Omega_M, \partial_a\Psi^A\}\frac{\partial\mathcal{L}}{\partial(\partial_a\Psi^A)} + \lambda^N\{\Omega_M, \Omega_N\}\,.
\end{aligned}
\tag{2.11}
$$

Although $\{\Omega_M, \dot{\Psi}^A\}$ is unknown because the replacement of $\dot{\Psi}^A$ by P_A is quite hard to be realized, the first term of equation (2.11) vanishes in accordance with equation (2.2). Consequently, this difficult task has been circumvented. Further constraints may be deduced by imposing the weak equality (2.9) whose integrand, i.e. the Poissson bracket of Ω_M and $\mathcal{H}$, is now available

$$\{\Omega_M, \mathcal{H}\} = \{\Omega_M, P_A\}\dot{\Psi}^A - \{\Omega_M, \Psi^A\}\frac{\partial \mathcal{L}}{\partial \Psi^A} - \{\Omega_M, \partial_a \Psi^A\}\frac{\partial \mathcal{L}}{\partial(\partial_a \Psi^A)} + \lambda^N\{\Omega_M, \Omega_N\}\,. \quad (2.12)$$

Equation (2.9), together with equation (2.12), is either identically satisfied, or it leads to determination of the Lagranger multipliers λ^M, or it implies existence of new constraints. In the last case the procedure has to be continued in a similar way until all the descendant (secondary, tertiary, *etc.*) constraints are finally derived. Note that it is only at the end of the process that one tries the substitution of velocities by momenta.

3. Hamiltonian analysis of A_P

With the modified hamiltonian procedure in mind we now begin to do the hamiltonian analysis for A_P. At first, we have to rewrite equation (1.1) explicitly in the expression composed of $\partial_0 X^\mu \partial_i X_\mu$ in order to calculate canonical momenta conjugate to X^μ, that is, we have to compute h^{ij} at first. To this end, we make the ADM reparametrization [15] of the induced metric h_{ij} in terms of a shift-vector N^a, a lapse function N and a p-metric γ_{ab}, all of which depend on ξ^i:

$$\begin{aligned} h_{00} &= -N^2 + \gamma_{ab}N^a N^b\,, \\ h_{0a} &= h_{a0} = \gamma_{ab}N^b\,, \\ h_{ab} &= \gamma_{ab}\,, \end{aligned} \quad (3.1)$$

and

$$\begin{aligned} h^{00} &= -N^{-2}\,, \\ h^{0a} &= h^{a0} = N^a N^{-2}\,, \\ h^{ab} &= \gamma^{ab} - N^a N^b N^{-2}\,, \end{aligned} \quad (3.2)$$

where γ^{ab} is the inverse of γ_{ab}. Substituting the definition of the induced metric equation (1.5) into equation (3.1), we solve N^a and N in terms of $\partial_0 X^\mu \partial_i X_\mu$:

$$\begin{aligned} N^a &= \gamma^{ab}(\dot{X}\partial_b X)\,, \\ N^2 &= -\dot{X}^2 + \gamma^{ab}(\dot{X}\partial_a X)(\dot{X}\partial_b X)\,, \end{aligned} \quad (3.3)$$

where the sum of lorentzian indices has been suppressed. Using equations (3.2) and (3.3) we therefore obtain the inverse of h_{ij} as follows:

$$\begin{aligned} h^{00} &= -\frac{1}{-\dot{X}^2 + \gamma^{ab}(\dot{X}\partial_a X)(\dot{X}\partial_b X)}\,, \\ h^{0a} &= h^{a0} = \frac{\gamma^{ab}(\dot{X}\partial_b X)}{-\dot{X}^2 + \gamma^{cd}(\dot{X}\partial_c X)(\dot{X}\partial_d X)}\,, \\ h^{ab} &= \gamma^{ab} - \frac{\gamma^{ac}\gamma^{bd}(\dot{X}\partial_c X)(\dot{X}\partial_d X)}{-\dot{X}^2 + \gamma^{ef}(\dot{X}\partial_e X)(\dot{X}\partial_f X)}\,. \end{aligned} \quad (3.4)$$

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Note that γ^{ab} is independent of $\partial_0 X^\mu$, and that $h^{ab} \neq \gamma^{ab}$ though $h_{ab} = \gamma_{ab}$ because h_{ij} is a $(p+1)$-metric while γ_{ab} a p-metric. With the help of equation (3.4) we re-formulate equation (1.1) to be

$$A_P = -\frac{T}{2}\int d^{p+1}\xi\sqrt{-g}\left[\frac{g_{00} - 2g_{0a}\gamma^{ab}(\dot{X}\partial_b X) + g_{ab}\gamma^{ac}\gamma^{bd}(\dot{X}\partial_c X)(\dot{X}\partial_d X)}{-\dot{X}^2 + \gamma^{ef}(\dot{X}\partial_e X)(\dot{X}\partial_f X)} - \right.$$
$$\left. - g_{ab}\gamma^{ab} + (p+3)\right]. \tag{3.5}$$

Equations (2.2) and (3.5) give the canonical momenta conjugate to g_{ij},

$$\Pi^{ij} \approx 0\,, \tag{3.6}$$

which mean $\frac{1}{2}(p+1)(p+2)$ primary constraints, and that conjugate to X^μ,

$$P_\mu = T\sqrt{-g}\left[-\dot{X}^2 + \gamma^{ab}(\dot{X}\partial_a X)(\dot{X}\partial_b X)\right]^{-2} \times$$
$$\times\left\{\left[-\dot{X}_\mu + \gamma^{ab}\partial_a X_\mu(\dot{X}\partial_b X)\right]\left[g_{00} - 2g_{0a}\gamma^{ab}(\dot{X}\partial_b X) + g_{ab}\gamma^{ac}\gamma^{bd}(\dot{X}\partial_c X)(\dot{X}\partial_d X)\right] - \right.$$
$$\left. - \left[-g_{0a}\gamma^{ab}\partial_b X_\mu + g_{ab}\gamma^{ac}\gamma^{bd}\partial_c X_\mu(\dot{X}\partial_d X)\right]\left[-\dot{X}^2 + \gamma^{ab}(\dot{X}\partial_a X)(\dot{X}\partial_b X)\right]\right\}, \tag{3.7}$$

respectively. It is obvious from equation (3.7) that to solve $\dot{X}^\mu$ in terms of P_μ is quite difficult and almost impossible. Following the procedure reviewed briefly in the above section, we now compute the time evolution of the primary constraints (3.6) by using equations (2.9), (2.12), and (3.5)

$$\dot{\Pi}^{ij} = -\frac{T}{2}\sqrt{-g}\left[\frac{1}{2}g^{ij}\left(-g_{kl}h^{kl} + p + 3\right) - h^{ij}\right] \approx 0\,, \tag{3.8}$$

from which we obtain the solution called sometimes an embedding relation

$$g_{ij} = h_{ij}\,. \tag{3.9}$$

When substituting equation (3.9) into equation (1.1), A_P turns to S_{NG}, which shows the classical equivalence between the two action as was discussed in detail in reference [2]. Here, however, in terms of 00 and $0a$ components of equation (3.9) we eliminate velocities in equation (3.7) and derive the following relations that in fact mean $p+1$ constraints associated with the invariance of A_P under $p+1$ reparametrizations

$$\left[g^{00}\left(g_{00} - g_{0a}g_{0b}\gamma^{ab}\right) + \frac{g_{0a}\gamma^{ab}(P\partial_b X)}{T\sqrt{-g}}\right]^2 = -\frac{1}{T^2 g g^{00}}\left[P^2 - \gamma^{ab}(P\partial_a X)(P\partial_b X)\right],$$
$$g_{0a} - g_{ab}g_{0c}\gamma^{bc} = -\frac{1}{g^{00}}\frac{(P\partial_a X)}{T\sqrt{-g}}\,. \tag{3.10}$$

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Before looking for further constraints let us classify these constraints into first- and second-classes. The first-class ones consist of 00 and $0a$ components of equation (3.6)

$$\begin{aligned} \Pi^{00} &\approx 0\,, \\ \Pi^{0a} &\approx 0\,, \end{aligned} \tag{3.11}$$

and equation (3.10) which can be reduced to the usual forms by using ab components of equation (3.9)

$$\begin{aligned} P^2 + T^2\gamma &\approx 0\,, \\ (P\partial_a X) &\approx 0\,, \end{aligned} \tag{3.12}$$

where $\gamma \equiv \det(\gamma_{ab})$. The second-class constraints contain ab components of equation (3.6) and of equation (3.9)

$$\begin{aligned} \Pi^{ab} &\approx 0, \\ g_{ab} - \gamma_{ab} &\approx 0\,. \end{aligned} \tag{3.13}$$

The number of independent degrees of freedom in phase space is given by the total number of degrees of freedom $2D + (p+1)(p+2)$ subtracted by twice of the number of first-class constraints $4(p+1)$ and by the number of second-class ones $p(p+1)$, that is, $2D - 2(p+1)$. This is the correct number, which shows that no more constraints exist. As a result, we find all the constraints by following the procedure introduced in section 2. In particular, the difficult task of replacing velocities by momenta in hamiltonian densities has been circumvented. The remaining procedure can be continued as dealt with, for instance, in references [16, 11] but is omitted in this and the next two sections as in reference [17].

4. Hamiltonian analysis of A_W^{I}

Substituting the ADM transformation (3.4) into equation (1.2), we rewrite A_W^{I} to be

$$\begin{aligned} A_W^{\mathrm{I}} = -\frac{T}{2}\int d^{p+1}\xi\sqrt{-g}\Bigg[&\frac{g_{00} - 2g_{0a}\gamma^{ab}(\dot{X}\partial_b X) + g_{ab}\gamma^{ac}\gamma^{bd}(\dot{X}\partial_c X)(\dot{X}\partial_d X)}{-\dot{X}^2 + \gamma^{ef}(\dot{X}\partial_e X)(\dot{X}\partial_f X)}\Phi^{(p+3)/2} - \\ &- g_{ab}\gamma^{ab}\Phi^{(p+3)/2} + (p+3)\Phi^{(p+1)/2}\Bigg]. \end{aligned} \tag{4.1}$$

The canonical momenta conjugate to g_{ij} take the same form as equation (3.6), and the momenta conjugate to Φ and to X^μ are given respectively by

$$\Pi_\Phi \approx 0\,, \tag{4.2}$$

which means a primary constraint, and

$$\begin{aligned} P_\mu = T\sqrt{-g}\Phi^{(p+3)/2}&\left[-\dot{X}^2 + \gamma^{ab}(\dot{X}\partial_a X)(\dot{X}\partial_b X)\right]^{-2} \times \\ \times\Big\{&\left[-\dot{X}_\mu + \gamma^{ab}\partial_a X_\mu(\dot{X}\partial_b X)\right]\left[g_{00} - 2g_{0a}\gamma^{ab}(\dot{X}\partial_b X) + g_{ab}\gamma^{ac}\gamma^{bd}(\dot{X}\partial_c X)(\dot{X}\partial_d X)\right] - \\ &- \left[-g_{0a}\gamma^{ab}\partial_b X_\mu + g_{ab}\gamma^{ac}\gamma^{bd}\partial_c X_\mu(\dot{X}\partial_d X)\right]\left[-\dot{X}^2 + \gamma^{ab}(\dot{X}\partial_a X)(\dot{X}\partial_b X)\right]\Big\}. \end{aligned} \tag{4.3}$$

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According to the modification of the hamiltonian procedure introduced in section 2, we calculate the time preservation of the primary constraints equations (3.6) and (4.2)

$$\dot{\Pi}^{ij} = -\frac{T}{2}\sqrt{-g}\Phi^{(p+1)/2}\left[\frac{1}{2}g^{ij}\left(-\Phi g_{kl}h^{kl}+p+3\right)-\Phi h^{ij}\right] \approx 0\,, \tag{4.4}$$

$$\dot{\Pi}_{\Phi} = -\frac{T(p+3)}{4}\sqrt{-g}\Phi^{(p-1)/2}\left(-\Phi g_{ij}h^{ij}+p+1\right) \approx 0\,. \tag{4.5}$$

From equation (4.5) we have

$$\Phi^{-1} = \frac{1}{p+1}g_{ij}h^{ij}\,, \tag{4.6}$$

and substituting equation (4.6) into equation (4.4) gives the embedding relation

$$g_{ij} = f(\xi)h_{ij}\,, \tag{4.7}$$

where $f(\xi)$ is an arbitrary function of ξ^i. The appearance of $f(\xi)$ in equation (4.7) shows that A_W^{I} possesses the Weyl symmetry as we mentioned before. When substituting equations (4.6) and (4.7) into equation (1.2), A_W^{I} reduces to S_{NG}. This expresses the classical equivalence between the two actions. Similar to the treatment in the above section, we eliminate velocities in equation (4.3) by using 00 and $0a$ components of equation (4.7) and then obtain the following $p+1$ constraints that associate with the invariance of the action A_W^{I} under $p+1$ reparametrizations

$$\begin{aligned}\left[g^{00}\left(g_{00}-f^{-1}g_{0a}g_{0b}\gamma^{ab}\right)+f^{-2}\Phi^{-(p+3)/2}\frac{g_{0a}\gamma^{ab}(P\partial_b X)}{T\sqrt{-g}}\right]^2 &= \\ = -f^{-3}\Phi^{-(p+3)}\frac{1}{T^2 g g^{00}}&\left[P^2-\gamma^{ab}(P\partial_a X)(P\partial_b X)\right],\\ g_{0a}-f^{-1}g_{ab}g_{0c}\gamma^{bc} = -f^{-1}\Phi^{-(p+3)/2}\frac{1}{g^{00}}\frac{(P\partial_a X)}{T\sqrt{-g}}\,.&\end{aligned} \tag{4.8}$$

Although equation (4.8) looks more complicated than equation (3.10), it reduces exactly to equation (3.12) after equation (4.6) and ab components of equation (4.7) are considered. Therefore, the system described by A_W^{I} contains the same set of first-class constraints as expressed by equations (3.11) and (3.12). As to second-class, from equations (3.6), (4.2), (4.6), and (4.7) we find that the corresponding set consists of the following $p(p+1)+2$ constraints

$$\Pi^{ab} \approx 0\,, \qquad g_{ab}-f\gamma_{ab} \approx 0\,, \tag{4.9}$$

$$\Pi_{\Phi} \approx 0\,, \qquad \Phi - f^{-1} \approx 0\,. \tag{4.10}$$

The number of independent degrees of freedom in phase space is $[2D+(p+1)(p+2)+2]-4(p+1)-[p(p+1)+2] = 2D-2(p+1)$ as expected.

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5. Hamiltonian analysis of $A_W^{\rm II}$

Substituting the ADM transformation (3.4) into equation (1.3), we rewrite $A_W^{\rm II}$ to be

$$A_W^{\rm II} = -T\int d^{p+1}\xi\sqrt{-g}\left[\frac{1}{p+1}\left(-\frac{g_{00}-2g_{0a}\gamma^{ab}(\dot X\partial_b X)+g_{ab}\gamma^{ac}\gamma^{bd}(\dot X\partial_c X)(\dot X\partial_d X)}{-\dot X^2+\gamma^{ef}(\dot X\partial_e X)(\dot X\partial_f X)}+\right.\right.$$
$$\left.\left.+\,g_{ab}\gamma^{ab}\right)\right]^{-(p+1)/2}. \qquad (5.1)$$

The canonical momenta conjugate to g_{ij} are given by equation (3.6) as before, and the momenta conjugate to X^μ, however, take the form

$$P_\mu = T\sqrt{-g}\left(\frac{1}{p+1}g_{ij}h^{ij}\right)^{-(p+3)/2}\left[-\dot X^2+\gamma^{ab}(\dot X\partial_a X)(\dot X\partial_b X)\right]^{-2}\times$$
$$\times\Big\{\left[-\dot X_\mu+\gamma^{ab}\partial_a X_\mu(\dot X\partial_b X)\right]\left[g_{00}-2g_{0a}\gamma^{ab}(\dot X\partial_b X)+g_{ab}\gamma^{ac}\gamma^{bd}(\dot X\partial_c X)(\dot X\partial_d X)\right]-$$
$$-\left[-g_{0a}\gamma^{ab}\partial_b X_\mu+g_{ab}\gamma^{ac}\gamma^{bd}\partial_c X_\mu(\dot X\partial_d X)\right]\left[-\dot X^2+\gamma^{ab}(\dot X\partial_a X)(\dot X\partial_b X)\right]\Big\}. \qquad (5.2)$$

With the modified hamiltonian procedure in mind we compute the time evolution of the primary constraints (3.6) by using equations (2.9), (2.12), and (5.1)

$$\dot\Pi^{ij} = -\frac{T}{2}\sqrt{-g}\left(\frac{1}{p+1}g_{kl}h^{kl}\right)^{-(p+3)/2}\left(\frac{1}{p+1}g^{ij}g_{kl}h^{kl}-h^{ij}\right)\approx 0\,, \qquad (5.3)$$

which leads to the same solution as equation (4.7). Similarly, the existence of an arbitrary function reveals the conformal invariance of $A_W^{\rm II}$. Substituting equation (4.7) into equation (1.3) makes $A_W^{\rm II}$ become S_{NG}, which gives the classical equivalence between the two actions. As treated in the above section we derive $p+1$ constraints from equation (5.2) by using equation (4.7)

$$\left[g^{00}\left(g_{00}-f^{-1}g_{0a}g_{0b}\gamma^{ab}\right)+f^{\frac{(p-1)}{2}}\frac{g_{0a}\gamma^{ab}(P\partial_b X)}{T\sqrt{-g}}\right]^2 = -\frac{f^p}{T^2 g g^{00}}\left[P^2-\gamma^{ab}(P\partial_a X)(P\partial_b X)\right]$$
$$g_{0a}-f^{-1}g_{ab}g_{0c}\gamma^{bc} = -f^{\frac{(p+1)}{2}}\frac{1}{g^{00}}\frac{(P\partial_a X)}{T\sqrt{-g}}\,, \qquad (5.4)$$

which are related with the invariance of the action $A_W^{\rm II}$ under $p+1$ reparametrizations. Again using ab components of equation (4.7) we can simplify equation (5.4) to be equation (3.12). That is, this system has the same set of first-class constraints as that of A_P and of $A_W^{\rm I}$. Its second-class set is nevertheless given by equation (4.9). The independent number of degrees of freedom in phase space is, of course, $2D-2(p+1)$ as we have known.

6. Conclusion

We have made hamiltonian analyses to A_P, $A_W^{\rm I}$, and $A_W^{\rm II}$ which, together with S_{NG}, constitute the new set of dual actions of bosonic p-branes. Because of the special definition of the contravariant components of the induced metric, we have utilized the ADM reparametrization to rewrite the actions explicitly in terms of $\partial_0 X^\mu \partial_i X_\mu$, and therefore we have been able to compute canonical momenta conjugate to X^μ defined as usual. What caused by such a definition of h^{ij} is that the actions contain highly non-linear terms composed of $\partial_0 X^\mu \partial_i X_\mu$, which makes it almost impossible to replace velocities by momenta in hamiltonian densities when following the usual canonical procedure. Fortunately, it has been discovered that such a replacement is not mandatory when canonically analysing the conformally invariant p-brane action S_W^{II}, and some modification to circumvent this difficult task has been suggested. What we have developed in this paper is that this modification of the hamiltonian analysis has a wider application that includes our newly constructed p-brane actions that contain higher non-linearity than that of S_W^{II}. Quite interesting is that the three new actions have the same first-class set of constraints, while their second-class sets are different from each other. The similar case also happens in the known p-brane actions S_P and S_W^{II}. Further studies, such as various possible supersymmetric extensions of the new actions and their canonical quantization, are now under consideration.

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